# $TM_{01p}$ TUBULAR AND CYLINDRICAL DIELECTRIC RESONATOR MODE

P. GUILLON, J.P. BALABAUD, Y. GARAULT
Laboratoire d'Electronique des Microondes
E.R.A. CNRS 535-U.E.R. des Sciences
123, rue A. Thomas 87060 LIMOGES Cedex -France-.

ABSTRACT

The electric dipole resonance of a cylindrical and a tubular dielectric resonator is investigated. Data are given about the resonant frequencies and the structure of the field surrounding the resonator. The data agree well with experimental results.

## Introduction

$TE_{01p}$ mode of dielectric cylindrical resonators has recently found a widespread application in various microwave devices like filters {1} or oscillators {2} .

But in many of these applications, to obtain better performances* and to reduce weight and size it is avantageous to use a cylindrical or a tubular dielectric resonator acting on its electric dipole mode($TM_{01p}$)

The $TM_{01p}$ cylindrical resonator mode is of interest because it has been shown {3} that it affords higher values for Q factor.

The dielectric tube cavity (consisting of a hollow circularly cylindrical tube of dielectric material suitably terminated at its ends by sufficiently large metal end walls) has been used in the past {4} as a device for millimeter and submillimeter waves.

The dielectric tube resonator that we have studied (that is without metal end walls) is very interesting, because the extra variable of wall thickness offers greatly flexibility of design, and might be expected to remove some of the compromise involved in the use of cylindrical resonator.

In this paper, we present rigourous approach to the problem of free oscillations and field pattern of $TM_{01p}$ mode of cylindrical and tubular dielectric resonators.

## Formulation of the problem

The analysis both available for shielded cylindrical (fig.1a) and tubular (fig. 1b) dielectric resonators consists to solve Helmothz equation expressed in cylindrical coordinates using finite difference method.

For these two geometries and for modes of revolution (modes without variation in θ ) Helmothz equation is :

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \left(4\pi\frac{f^2}{C^2}\varepsilon_r - \frac{1}{r^2}\right)\psi + \frac{\partial^2\psi}{\partial z^2} = 0 \quad \ldots\ 1$$

f : resonant frequency of the considered mode

$\psi = H_\theta$ for TM modes (For TE modes $\psi = E_\theta$ ).

For obtaining the resonant frequencies and field pattern of electric dipole mode of cylindrical and tubular resonators, we solve equation {1} be means of finite difference method.

For that we introduce N x M nodes (i,j) in the cross section of the system shown in fig.2. Assuming usual finite difference approximation {7} , {1} is written :

$$\{\psi_{i+1,j} + \psi_{i-1,j} - 2\psi_{i,j}\}\frac{1}{\Delta r^2} + \{\psi_{i+1,j} - \psi_{i-1,j}\}\frac{2r}{\Delta r}$$
$$+ \{4\pi^2\frac{f^2}{c^2}\varepsilon_r - \frac{1}{r^2}\}\psi_{ij} + \{\psi_{i,j+1} + \psi_{i,j-1} - 2\psi_{ij}\}\frac{1}{\Delta z^2} = 0 \quad \ldots\ 2$$

This equation (2) is applicable to any node (i,j) in the dielectric medium of permittivity $\varepsilon_r$.

At the interfaces between dielectric mediums we have to satisfy the continuity of the tangential components of E.M. fields.

- Interface parallel to z axis :

$$\frac{1}{\varepsilon_r}\{\frac{1}{r}H_\theta + \frac{\partial H_\theta}{\partial r}\}\varepsilon_r = \frac{1}{\varepsilon_a}\{\frac{1}{r}H_\theta + \frac{\partial H_\theta}{\partial r}\}\varepsilon_a \quad \ldots\ 3$$

- Interface parallel to r axis :

$$\frac{1}{\varepsilon_r}\{\frac{\partial H_\theta}{\partial z}\}\varepsilon_r = \frac{1}{\varepsilon_a}\{\frac{\partial H_\theta}{\partial z}\}\varepsilon_a \quad \ldots\ 4$$

We note also that these boundaries equations should be modified at the corners {7} (shaded zones in fig.1) and on the metallic walls where they satisfy short circuit conditions.

At all points (i,j) defined in the cross section of the system the corresponding equation give a linear system.

$$A\,\psi = \lambda\,\psi \quad \ldots\ 5$$

A band matrix non symmetric, λ the eigenvalue.

By means of a computer we can determinate :

- the eigenvalues λ which give the resonant frequency (f) of the system

$$f = 2\pi\sqrt{\frac{1}{\lambda . C^2}}\ \frac{\Delta r}{\Delta z} \quad \ldots\ 6$$

- the eigenvectors associated to the matrix A which permit to evaluate at any node of the structure the value of $\psi = H_\theta$ . By applying Maxwell equations we can also determine the other components $E_r$ and $E_z$ of the E.M. field.

1. $TM_{01p}$ mode of the cylindrical dielectric resonator.

The theoretical and experimental variations of the resonant frequencies of the $TM_{01p}$ mode of the cylindrical dielectric resonator as a function of metallic box dimensions are given in fig.3. These curves have been obtained using a $Ba_2\ Ti_9\ O_{20}$ temperature stabilised resonator of permittivity $\varepsilon_r = 35$. Of great interest to the problem of coupling is the field pattern of this electric dipole mode. Typical E field plots both field distribution associated with resonance are given respectively in fig. 4 and fig.5.

2. $TM_{01p}$ mode of the tubular dielectric resonator

- mode charts

Fig. 6, 7, 8 illustrate respectively for $Ba_2\ Ti_9\ O_{20}$ tubular dielectric resonator.

- the change in resonance frequency as a function of the ratio of the cylinder diameter (D'/D) and height (H'/H)
- for a fixed height of the resonator and the box, the diminishing of the resonant frequency as a function of the cylinder diameter ratio (D'/D).
- for a fixed diameter of the resonator and the box, the increasing of the resonant frequency as a function of the height ratio (H'/H).
- the value of the resonant frequency of $TM_{011}$ modes of a tubular cavity similar to that used in publications 4 ($H_1 = 0$).

- field maps

The knowledge of the relative field distribution is always very desirable for using these resonators in microwave devices.

The graph 9 presents some of the more interesting change in the $TM_{01p}$ field. In this field map only the radial dependance of the field expression has been plotted (note that the z dependance can be obtained identically).

On fig. 10 we present an example of the normalized tangents to the E line field of the electric dipole mode.

## Conclusion

A rigourous analysis of resonant modes of cylindrical and tubular dielectric resonator has been presented We can note that this analysis is also applicable to either revolution modes of TE and TM types.

It should finally be remarked that the $TM_{01p}$ mode of both cylindrical and tubular resonator will be used in microwave devices because it permits to obtain better performances than the magnetic dipolar mode of the cylindrical resonator.

## References


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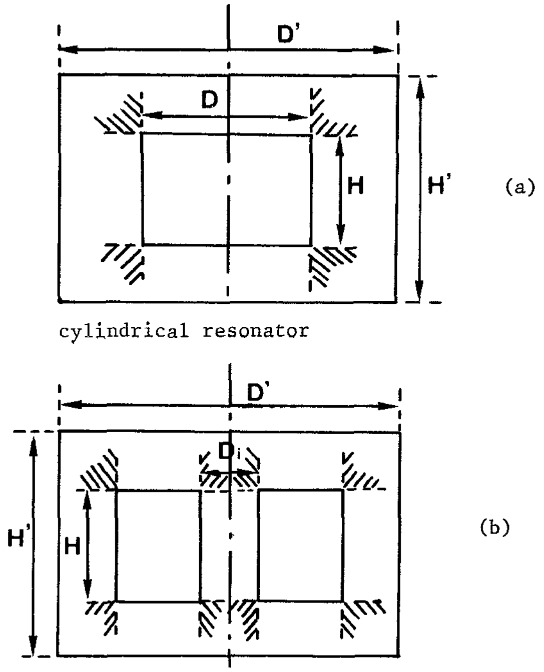
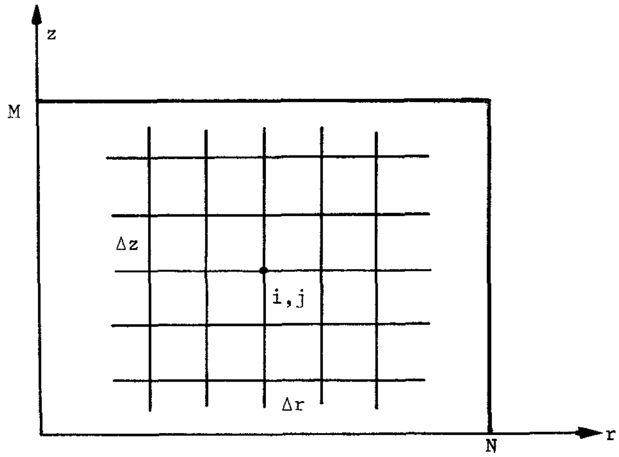



cylindrical resonator



tubular resonator

- Figure 1 -




- Figure 2 -

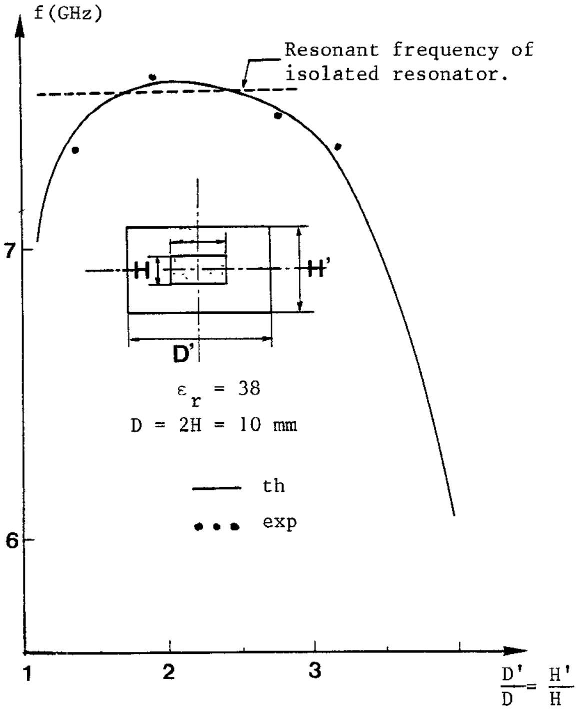



Fig.3 : $TM_{01p}$ mode of cylindrical resonator.

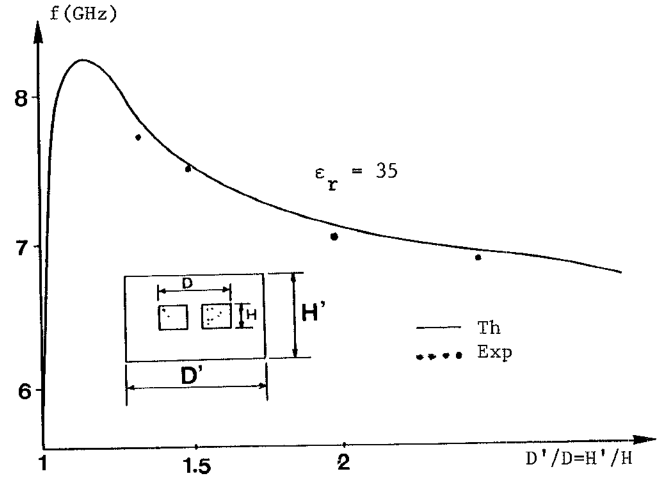



Fig.6 = $TM_{01p}$ mode of tubular dielectric resonator.

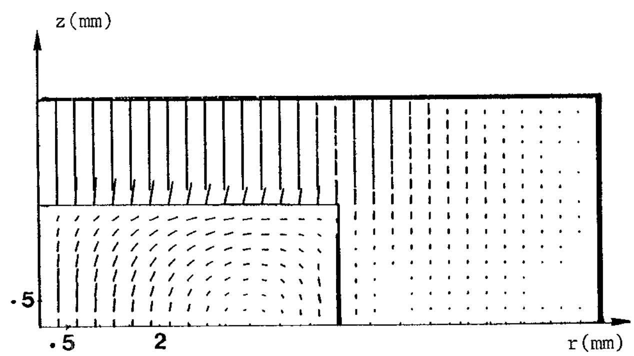



Fig.4 : $TM_{01p}$ mode of cylindrical resonator : tangent to E field lines.

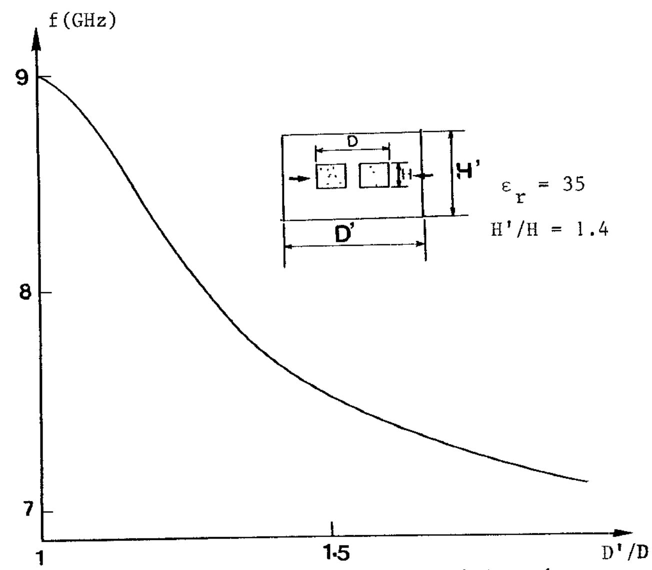



Fig.7 : $TM_{01p}$ mode of the tubular dielectric resonator.

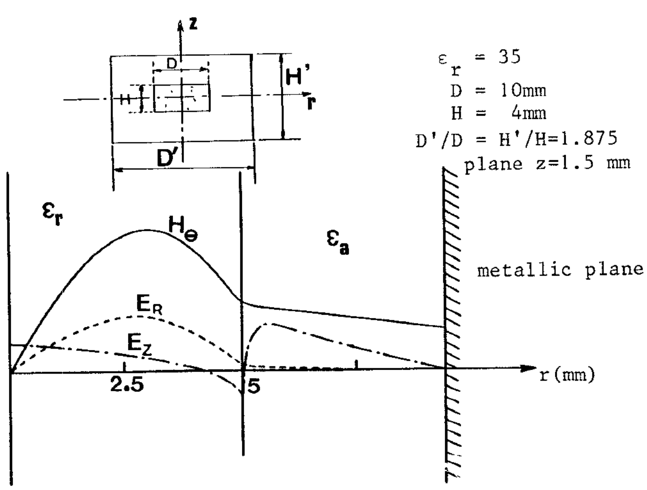



Fig.5 : $TM_{01p}$ cylindrical resonator mode.

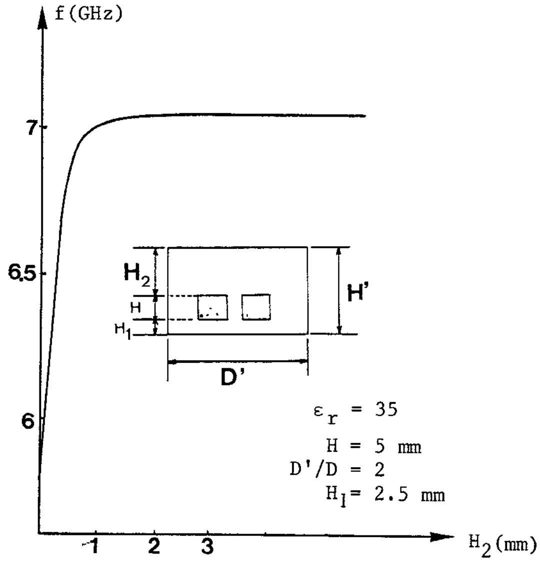



Fig.8 : $TM_{01p}$ mode of tubular dielectric resonator.

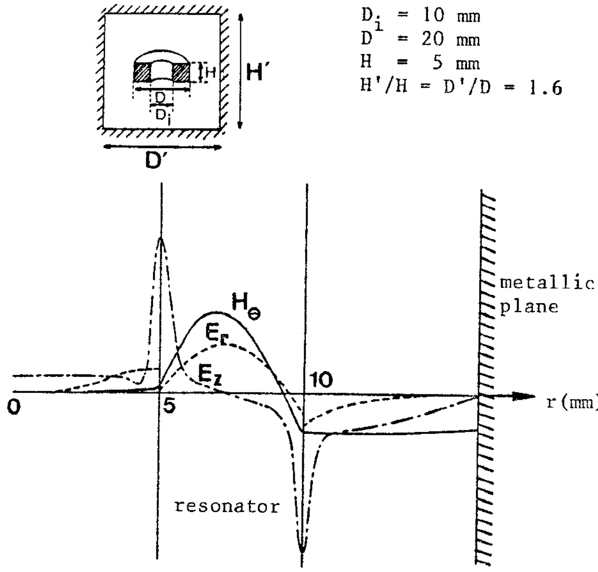



Fig.10 : Tubular dielectric resonator.

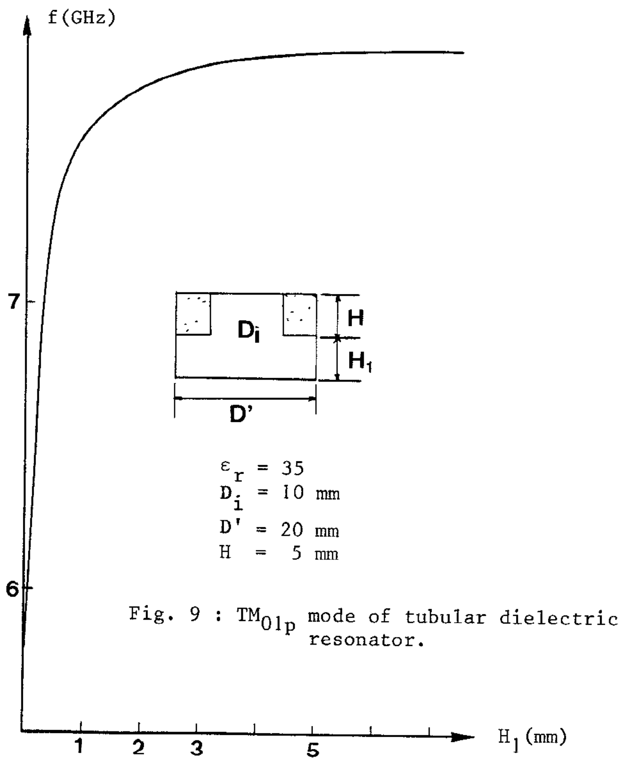



Fig.9 : $TM_{01p}$ mode of tubular dielectric resonator

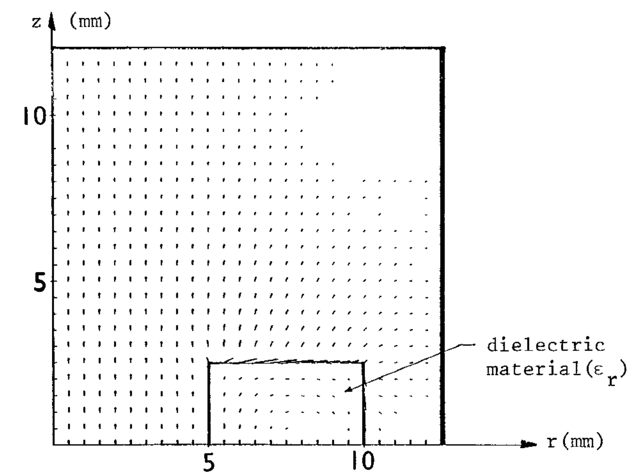



Fig.11 : $TM_{01p}$ mode of tubular dielectric resonator : tangent to E field lines.